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THE RATES OF PRODUCTION AND LOSS OF ELECTRONS IN THE F REGION OF THE IONOSPHERE

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CONTENTS

In the preceding paper by Schmerling & Thomas (1956) it was shown how, from experimental $h'(f)$ records, it is possible to deduce the electron distributions in the F layer appropriate to an average magnetically quiet day in any one month; and some account was given of these distributions at Slough, Huancayo and Watheroo for different times of day, seasons and epochs in the solar cycle. In this paper these distributions are used as experimental facts from which the rates of production and loss of electrons are deduced. Use is also made of electron distributions determined, in rapid succession, near sunrise at Cambridge.

It is here recognized that there may be important vertical movements of the electrons in the F region, but no assumptions are made about their magnitudes. Methods of analysis are used which minimize their effects.

In part I it is shown that the behaviour of the quiet F layer, above 240 km, at the three places mentioned, at all times of the day, the year, and the solar cycle, is more consistent with the supposition that the rate of loss of electrons is given by $-\mathrm{d}N/\mathrm{d}t = K_1 N$ rather than by $-\mathrm{d}N/\mathrm{d}t = K_2 N^2$. Between heights of about 250 and 350 km the loss coefficient K_1 seems to vary with height as given approximately by

$$K_1 = 10^{-4} \exp\left\{\frac{300-h\ (\mathrm{km})}{50}\right\}\ \mathrm{s}^{-1}.$$

The significance of this type of loss coefficient is discussed.

In part II it is realized that, if the above-mentioned loss coefficient were the correct one, then, because it decreases rapidly upwards, it could give rise to a peak of electron density considerably above the peak of electron production. This leads us to consider in detail the hypothesis of Bradbury (1938) that the F_1 and F_2 layers are both produced by the same ionizing radiation, acting on the same atmospheric constituent, and that the two peaks of electron density are caused by a

suitable height variation of the loss coefficient. This hypothesis is discussed critically, in relation to the experimental results, and is shown to be self-consistent.

It is shown that, if Bradbury's hypothesis is accepted as correct, the scale height of the ionizable constituent between 180 and 350 km is about 45 km. This is in better agreement with the *R* model of the upper atmosphere deduced by Bates (1954) from the results of rocket experiments, than with the *G* model deduced from the results of experiments made on the ground. It is also shown how the experimental results lead to the deduction of limits to the movements caused by the diffusion of electrons under gravity. The movements expected on Bates's *R* model are greater than those deduced from the experiments.

Although the mechanisms of electron loss and production discussed in this paper seem to fit the experimental facts there might be others, of more or less complication, which would also be self-consistent and would satisfy the tests we have used. Much more work is required, on world-wide results, before an explanation can be claimed to be unique.

1. Introduction

In the preceding paper Schmerling & Thomas (1956) have deduced the electron density at a series of different heights and times of day at Huancayo, Watheroo and Slough, at midsummer, midwinter and equinox and at different epochs of the solar cycle. They restricted their attention to the international magnetically quiet days in the appropriate months, and from them they deduced the form of the 'mean quiet *F* layer' which gave a description of the fundamental behaviour of the layer without undue emphasis on those small-scale movements which differ from day to day. It is the purpose of this paper to use these results to make some deductions about the rates of production and loss of the electrons.

It is now recognized, thanks largely to the work of Martyn (1955*a*), that regular movements play an important part in determining the electron distribution in the *F* layer, and, until their nature is known, it is difficult to deduce the rates of production and loss of electrons. In this paper we shall not make assumptions about the detailed nature of these movements; we shall, instead, use methods of analysis which, while recognizing their presence, minimize their effects. We shall then neglect these effects altogether, and shall find that our conclusions, based on a considerable mass of data, are consistent. We shall then suggest that we have arrived at a possible description of the rates of production and loss of electrons in the *F* layer. Our method of analysis is such that we cannot claim that the suggested mechanism is unique; there might be others which would satisfy our tests, but so far we have not found any.

It has been alternatively suggested in the past that electrons, of number density N, might disappear from the *F* region of the ionosphere by a process analogous to recombination, in which the rate is proportional to N^2, or by one analogous to attachment, in which the rate is proportional to N. It has also been suggested that the constant of proportionality might be smaller at greater heights. Some of these possibilities have been discussed by Bates & Massey (1948). In part I of this paper these suggestions will be considered in the light of the observed facts.

There have in the past been two different hypotheses about the rate of production of electrons in the *F* region. In both it is supposed that the electrons are produced by the ionization of some atmospheric constituent by ultra-violet radiation from the sun. The two different suggestions are concerned with explaining why an F_2 layer is formed above, and distinct from, the F_1 layer. In one suggestion it is assumed that the peak of electrons in the

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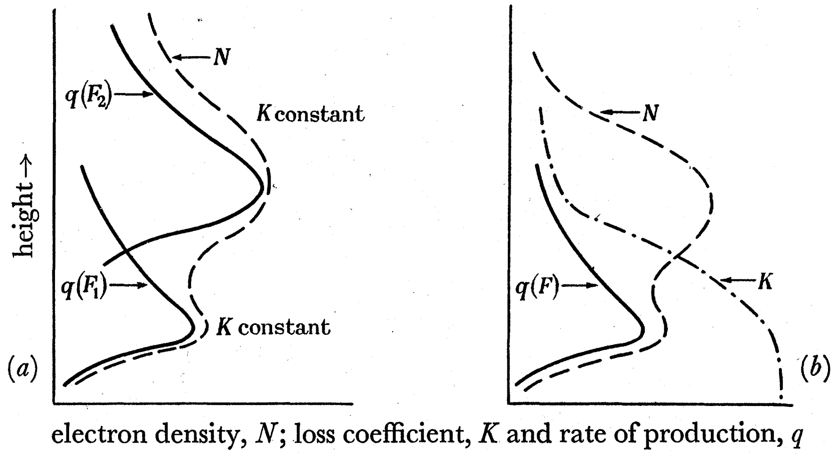
F_2 layer is located near the peak of production of electrons, and that a second, and independent, peak of production is responsible, lower down, for the formation of the F_1 layer. On this hypothesis it is supposed that both the layers are similar* to classical Chapman (1931 *a*) layers. We shall call this hypothesis for the formation of the F_2 layer the 'hypothesis of a Chapman layer'.

On the second hypothesis the peaks of electron density in the F_2 and F_1 layers are supposed both to be produced by the same incident radiation acting on the same atmospheric constituent. The peak of electron production is supposed to be near the level of the F_1 layer, but the rate of electron loss decreases so rapidly above that level that a second peak of electron density is formed and constitutes the F_2 layer. This suggestion was first made by Bradbury (1938) and has later been repeated by several other workers. We shall call this the 'Bradbury hypothesis' for the F_2 layer. The two hypotheses are illustrated schematically in figure 1.



(*a*) Chapman (*b*) Bradbury

FIGURE 1. To illustrate the 'Chapman' and the 'Bradbury' hypotheses for the formation of the F_2 layer.

In part II of this paper we shall discuss Bradbury's hypothesis in the light of observed facts in an attempt to see how far it can be made self-consistent.

The conclusions of the paper are collected and discussed in part III.

PART I. THE RATE OF LOSS OF ELECTRONS

2. METHODS OF CALCULATION

In the past several attempts (Appleton 1937; Seaton 1947, Baral & Mitra 1950), have been made to determine the rates of electron loss at different heights in the F region, but it is now generally accepted that the results are vitiated by the occurrence of movements. In this paper we shall use two different methods of calculation in which it is suggested that the effects of movements need not be included. In one method (§3) use is made only of night-time results and in the other (§§4 to 7) results obtained over 24 h are used. The former

* More elaborate hypotheses have allowed for height changes of scale height and coefficient of electron loss, but the essential feature, that the two layers are produced by independent processes, with their production peaks at different levels, is not, on these hypotheses, altered.

provides evidence that the loss coefficient is smaller at greater heights, and the latter leads us to the conclusion that, between 240 and 350 km, the loss rate is proportional to the electron density N.

3. Results at night

By making use of the results of Schmerling & Thomas (1956) it is possible to consider the variations of electron density at any one of a series of heights. The results from Slough and Watheroo for the different seasons in years of high sunspot number show that, at all heights, at night the electron density decreased uniformly with time as though controlled by a process

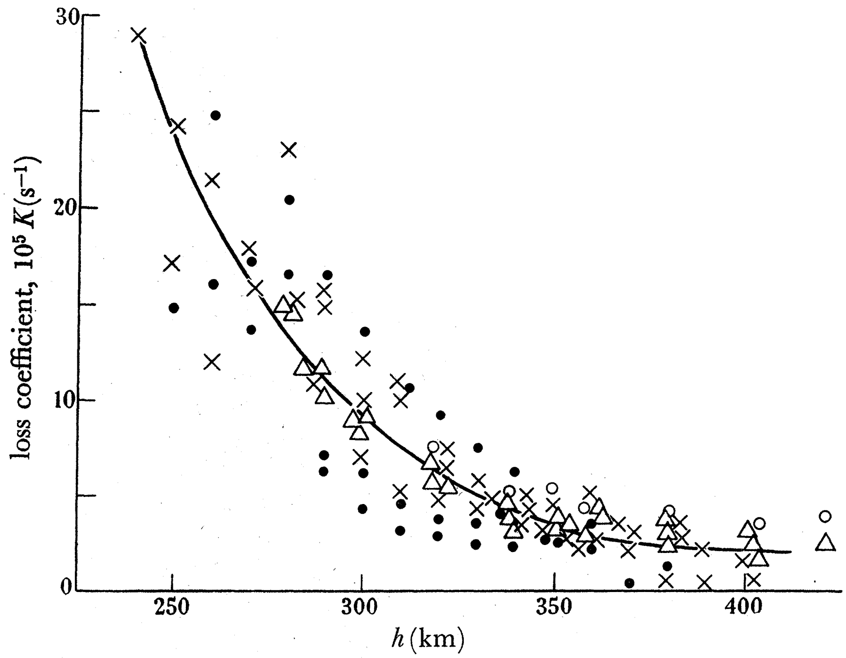


Figure 2. Loss coefficient K determined from night-time observations in years of high sunspot number. × average from the 'mean quiet F layer' for three seasons from Slough; ● average from the 'mean quiet F layer' for three seasons from Watheroo; △ average for forty individual nights in different seasons at Slough; ○ values of K for three nights at Fraserburgh; — mean curve through observations.

of electron loss. At Huancayo, and at Slough and Watheroo in years of low sunspot number, the behaviour was not so simple, and the electron density at some levels increased during some part of the night. We propose to consider the simpler results from Slough and Watheroo in years of high sunspot number and to see whether they lead to consistent conclusions on the supposition that at those times the effects of movements were negligible compared with the effects of electron loss.

The $N(t)$ curves, which showed how the electron density (N) varied with time (t) at a series of given heights, were examined. It was found that the results agreed fairly well with the assumption that the loss coefficient at each height remained constant for some 8 h after sunset. The accuracy was not sufficient to enable us to decide with certainty whether a rate of loss proportional to N^2 or to N fitted the results better. We give only the values appropriate to a rate of loss KN, since this is what we shall be concerned with later. The values of K, deduced for a series of different heights, from the 'mean quiet F layers' appropriate to several months at Slough and Watheroo, are shown in figure 2.

We have also examined the results obtained on some individual nights by selecting those occasions on which the electron density decreased smoothly at all levels. The results for three separate nights at Fraserburgh and the average for 40 nights at Slough are included in the figure.

Although the results shown in figure 2 are derived from several places and seasons they are, nevertheless, fairly consistent. This fact encourages us to consider the possibility that they do, in fact, represent the loss coefficient, and that on these occasions movements had little effect. There are, of course, movements which, together with different loss coefficients, would lead to the experimental results, but these would have to have the same effects at all places and seasons considered. We shall, for the time being, explore the results of supposing that figure 2 correctly represents the magnitude of the loss coefficient at different heights.

Between the heights of 250 and 350 km the results of figure 2 are fitted fairly closely by the curve which is represented by

$$K = 10^{-4} \exp\left\{\frac{300-h\,(\mathrm{km})}{50}\right\}\ \mathrm{s}^{-1}. \tag{1}$$

The possible significance of this result is discussed in §15 (c).

4. The second method of investigation

We first assume that the behaviour of the 'mean quiet F layer' described in §1 would be repeated from one day to the next. The total number of electrons produced in 24 h in a unit column of the atmosphere would then be equal to the total number disappearing by processes of electron loss, and this equality would be maintained whatever vertical movements there might be. To make use of this necessary equality we shall calculate: (i) the total number produced on some hypothesis (p) about their rate of production, and (ii) the total number lost, by considering the experimentally determined values of N, together with some hypothesis (l) about the rate of loss. In our different forms of the hypothesis (p) we shall make assumptions about how the production varies with height, time and geographical location; and in different forms of (l) we shall make assumptions about how the loss rate depends on the electron density and the height. The absolute magnitudes of the production and loss rates will not be taken as known, but we shall assume that the loss rate is the same at different places and times. We shall then, for different seasons and places, at the same solar epoch, determine the ratio of the total number of electrons produced in 24 h to the total number lost, and shall seek for that pair of hypotheses (p) and (l) for which the ratio is most nearly constant. We shall then suggest that this pair corresponds most nearly to reality.

Unfortunately, it is not possible to consider the whole extent of a unit column from the ground to the outside of the atmosphere. We shall, instead, consider the unit column to be terminated below at a height of 240 km, and above at the height of the F_2 electron peak. It then becomes necessary to consider the transfer of electrons across these two boundary planes, in the following way.

At a fixed height the rate of change of electron density is given by

$$\mathrm{d}N/\mathrm{d}t = q - K_n g(h)\, N^n - d(Nw)/\mathrm{d}h, \tag{2}$$

where q represents the rate of production, and w represents the vertical velocity of the electrons. The loss rate is here written $K_n g(h) N^n$, so that, by giving n the values 1 or 2, it can be assumed proportional either to N or to N^2, and the function $g(h)$ allows for the possibility that the loss coefficient might change with height.

Now let us consider a unit column extending from height h_1 to height h_2, and let us integrate equation (2) with respect to height and with respect to time from t_1 to t_2 to give

$$\left[\int_{h_1}^{h_2} N\,\mathrm{d}h\right]_{t_1}^{t_2} = \int_{t_1}^{t_2}\int_{h_1}^{h_2} q\,\mathrm{d}h\,\mathrm{d}t - \int_{t_1}^{t_2}\int_{h_1}^{h_2} K_n g(h)\, N^n\,\mathrm{d}h\,\mathrm{d}t - \left[\int_{t_1}^{t_2} (Nw)\,\mathrm{d}t\right]_{h_1}^{h_2}. \tag{3}$$

If $t_2 = t_1 + 24\,\mathrm{h}$ the term on the left-hand side is zero for a distribution of electrons which repeats from day to day. The first two terms on the right-hand side are the total number produced and the total number removed by the loss process in 24 h, and they would be equal if the last term, which represents the effect of the movements, were zero. Now, although the movement term in equation (2) is known to be of such importance that its neglect cannot, in general, lead to useful results, it appears to be worth exploring the consequences of neglecting the movement term in equation (3). Some justification for this procedure might be found in the fact that most theories of the movement suggest that the velocity (w) is oscillatory and that it is directed upwards and downwards for equal times during 24 h. If this were so the movement term in equation (3) could well be relatively less than that in equation (2), and it might be small in comparison with the other terms. It should be noticed that, in our calculations, the upper limit will be taken at the height of the F_2 electron peak, but in equation (3) it was taken at the constant height h_2. This difference does not invalidate our reasons for suggesting that the movement term in equation (3) might be small.

The procedure here is to neglect the movement term in equation (3) and to see whether any pair of hypotheses (p) and (l) provides results which are more consistent than other pairs. This is not the same as neglecting the effects of movements in re-distributing the electrons: the movement term in equation (2) could be quite large. The method of calculation is as follows.

The rate of production of electrons in unit volume at height h and time t can be represented by $q_0 f(h, t)$, where q_0 is the rate at the production peak when the sun is in the zenith. q_0 is expected to vary with the epoch in the solar cycle. The height of the production peak will depend on the chosen hypothesis (p) and will be different according as the F_2 layer is supposed to be Chapman-like or Bradbury-like. The function $f(h, t)$ will depend on the geographical location and the season. The total number of electrons produced, between heights h_1 and h_2 in unit column in 24 h is then given by $q_0 P$, where

$$q_0 P = q_0 \int_0^{24}\int_{h_1}^{h_2} f(h, t)\,\mathrm{d}h\,\mathrm{d}t \tag{4}$$

The magnitude of P (for production) can be evaluated for different hypotheses, places and seasons.

As previously explained we shall allow for different hypotheses about the rate of loss of electrons by writing

$$-\mathrm{d}\{N(h,t)\}/\mathrm{d}t = K_n g(h)\,\{N(h,t)\}^n, \tag{5}$$

in which n takes the value 1 or 2. The total number of electrons lost, in 24 h, in unit column between heights h_1 and h_2 is then given by $K_n L$, where

$$K_n L = \int_0^{24}\int_{h_1}^{h_2} -[\mathrm{d}\{N(h,t)\}/\mathrm{d}t]\,\mathrm{d}h\,\mathrm{d}t$$

$$= K_n \int_0^{24}\int_{h_1}^{h_2} g(h)\,\{N(h,t)\}^n\,\mathrm{d}h\,\mathrm{d}t. \tag{6}$$

The magnitude of L (for loss) can be evaluated numerically from observed values of $N(h,t)$ and assumed forms of $g(h)$.

The total number of electrons produced in the bounded unit column in 24 h (equation (4)) may now be equated to the total lost (equation (6)), to give the relation

$$q_0/K_n = L/P. \tag{7}$$

The quantities P and L are evaluated from equations (4) and (6) for a series of different seasons and places at epochs when the sunspot numbers are nearly the same. The places are chosen so that P and L vary over a wide range.* Equation (7) shows that the ratio L/P should be constant, and equal to q_0/K_n. We therefore chose that pair of hypotheses to be the most satisfactory for which the ratio L/P is found to be most nearly constant. The calculation of P and L is discussed in §5 and 6 and the best hypotheses are selected in §7.

5. The electrons produced in 24 hours

It is assumed that the electrons are produced in a gas of constant scale height H by the absorption of a monochromatic radiation, so that $q_0 f(h,t)$ in equation (4) is given by the expression due to Chapman (1931 *a*)

$$q = q_0 f(h,t) = q_0 \exp\{1 - z - \mathrm{e}^{-z}\sec\chi(t)\}, \tag{8}$$

where $\chi(t)$ represents the sun's zenith distance $z = (h-h_0)/H$, where h_0 is the level at which q has a maximum when $\chi = 0$. It can then be shown that

$$\int_{h_1}^{h_2} q_0 f(h,t)\,\mathrm{d}h = q_0 H \int_{z_1}^{z_2} f(z,t)\,\mathrm{d}z$$

$$= q_0 H \cos\chi(t)\,[\exp\{1-\mathrm{e}^{-z_2}\sec\chi(t)\} - \exp\{1-\mathrm{e}^{-z_1}\sec\chi(t)\}]. \qquad (9)\dagger$$

We now consider two different hypotheses (p) as follows.

(*a*) *The hypothesis of a Chapman region*

In this hypothesis it is assumed that the peak of the electron density in the F_2 layer is near the peak of the corresponding electron production, and the electrons which can be detected are assumed to be all those produced below the peak of production. The upper limit in equation (9) is therefore $z_2 = z_m$, where z_m is given by $\mathrm{e}^{z_m} = \sec\chi$ by standard theory, so that, from equation (9)

$$\int_{h_1}^{h_2} q_0 f(h,t)\,\mathrm{d}h = q_0 H \cos\chi(t)\,[1 - \exp\{1 - \mathrm{e}^{-z_1}\sec\chi(t)\}].$$

* P and L change with the solar epoch, the season, and the geographical location. The values of $N(h)$ from which L is deduced vary even more radically.

† If we put $z_1 = -\infty$ and $z_2 = +\infty$ we see that this expression is equal to $q_0 He \cos\chi$ so that the rate of production of electrons in a unit column extending right through the atmosphere is proportional to $\cos\chi$, as is well known.

If the integration were extended down to the ground ($z_1 = -\infty$) the magnitude would be $q_0 H \cos\chi$. If it is extended only down to $z_1 = -2$ the magnitude is

$$q_0 H \cos\chi\{1 - \exp(1 - 7{\cdot}4 \sec\chi)\},$$

which is, for all values of χ, sufficiently nearly the same. Thus if we concern ourselves with a column extending from the production peak down to levels two scale heights or more below that peak, then the rate of production of electrons in this column is $q_0 H \cos\chi$. The total number of electrons produced in this column in 24 h is then (equation (4)) given by $q_0 P_C$, where

$$q_0 P_C = q_0 H \int_0^{24} \cos\chi(t)\, \mathrm{d}t,$$

so that

$$P_C/H = \int_0^{24} \cos\chi(t)\, \mathrm{d}t. \tag{10}$$

Here we write P_C to indicate that the quantity is appropriate to the hypothesis of a Chapman region.

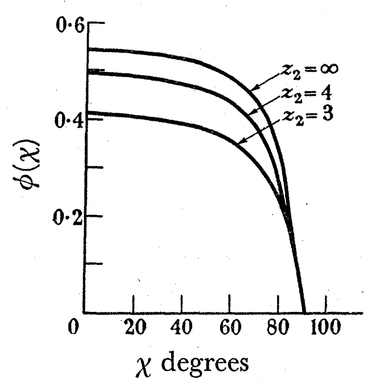


FIGURE 3. The function $\phi(\chi)$ of equation (12) for different values of z_2.

The magnitude of P_C/H is evaluated for any place and any month by calculating $\cos\chi(t)$ as a function of time in the usual way and performing a graphical integration. It is listed, for different months at Slough, Watheroo, and Huancayo, in line 4 of table 1.

(*b*) *The Bradbury hypothesis*

We next assume that the electrons in the F_2 layer are produced by the radiation which causes a peak of production in the F_1 layer. We shall consider the production and loss of electrons throughout a unit column extending from the peak of electron density in F_2 downwards to a level which is one and a half scale heights* (H) above the peak of production in the F_1 layer. If the peak of the F_2 electron distribution is at a reduced height z_2 above the peak of production, then equation (9) shows that the rate of production of electrons throughout the column considered is

$$q_0 H \int_{1\cdot5}^{z_2} f(z, t)\, \mathrm{d}z = q_0 H \phi(\chi), \tag{11}$$

where

$$\phi(\chi) = \cos\chi\,\{\exp(1 - \mathrm{e}^{-z_2} \sec\chi) - \exp(1 - \mathrm{e}^{-1\cdot5} \sec\chi)\}. \tag{12}$$

Figure 3 shows the form of $\phi(\chi)$ when $z_2 = 3$, 4 and ∞. In the experimental data which we shall use the electron peak was at a height between 280 and 380 km. To obtain preliminary

* In §15(*b*) we shall justify the use of $z_1 = 1{\cdot}5$ at a height of 240 km.

results we shall suppose that it is about 4 scale heights above the production peak, and shall use the values of $\phi(\chi)$ for $z_2 = 4$. We shall show later that this choice is reasonable. The extra complication involved in trying to choose the best value of z_2 for each particular case does not seem to be justified, so long as the magnitude of H remains uncertain.

The total number of electrons produced, in 24 h, in unit column extending from $z_1 = 1{\cdot}5$ to $z_2 = 4$ is thus given by $q_0 P_B$, where, from equations (4) and (11),

$$q_0 P_B = q_0 H \int_0^{24} \phi\{\chi(t)\}\,\mathrm{d}t$$

or

$$P_B/H = \int_0^{24} \phi\{\chi(t)\}\,\mathrm{d}t. \tag{13}$$

Here we write P_B to indicate that the quantity is appropriate to the hypothesis of a Bradbury layer.

The magnitude of P_B/H is evaluated for any place and season by calculating $\phi\{\chi(t)\}$ as a function of time with the help of equation (12), with z_2 put equal to 4, and performing a numerical integration. Values are shown, appropriate to Slough, Huancayo and Watheroo, at different times of the year in line 5 of table 1.

6. The electrons lost in 24 hours

The results of § 3 appear to show that the loss coefficient decreases with increasing height so that the F_2 layer could be produced by the Bradbury process. For the sake of completeness we shall, nevertheless, consider what would happen if the loss coefficient were independent of height, so that the F_2 layer would be of the Chapman type. For this purpose we put $g(h)$ in equation (5) equal to unity and the equation then takes one or other of the forms

$$-\mathrm{d}\{N(h,t)\}/\mathrm{d}t = K_1 N(h,t), \tag{14}$$

$$-\mathrm{d}\{N(h,t)\}/\mathrm{d}t = K_2\{N(h,t)\}^2. \tag{15}$$

If the F_2 layer is a Bradbury layer the loss coefficient must decrease with increasing height. As a working hypothesis we shall assume that, at the heights with which we are concerned, $g(h) = \exp\{(300-h)/50\,(\mathrm{km})\}$ in accordance with equation (1). In order to determine the total number of electrons lost in 24 h it is then necessary to evaluate the quantity L defined by equation (6) as

$$K_n L = K_n \int_0^{24}\int_{h_1}^{h_2} g(h)\,\{N(h,t)\}^n\,\mathrm{d}h\,\mathrm{d}t. \tag{16}$$

The function $g(h)$ has been taken as unity at a height of 300 km, so that K_n represents the magnitude of the loss coefficient at that height. Values of $N(h,t)$ were taken from the 'mean quiet F layers' computed by Schmerling & Thomas (1956). Their results were used in the form of curves showing $N(t)$ at a series of heights separated by intervals of 20 km. The integral of equation (16) was then computed numerically. The lower limit h_1 was taken as 240 km, and the upper limit at any one time was the greatest height to which N could be measured, and corresponded to the height of the electron peak. This limit varied through the day but was usually between 280 and 350 km.

The magnitudes of L, for the different seasons and places, calculated on the different hypotheses of equations (14) and (15), are listed in lines 6, 7, 8 and 9 of table 1, where they are labelled $L(K_1)$ and $L(K_2)$ for the case where $g = 1$ and $L(K_1 g)$ and $L(K_2 g)$ for the case where g varied with height.

TABLE 1

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1 mean sunspot number $\bar{R}$	0	0	0	10	10	10	20	20	20	mean of cols. 1 to 9	coeff. of variation of col. 10	60	60	80	80	90	100	100	100	mean of cols. 12 to 19	coeff. of variation of col. 20	
2 place	S	W	H	S	H	W	S	W	H			W	H	H	W	S	S	S	H			
3 month	Dec. 1953	June 1944	June 1944	Sept. 1953	Mar. 1944	Mar. 1944	June 1953	Dec. 1944	Dec. 1944			Mar. 1939	Mar. 1939	June 1940	June 1940	July 1950	Jan. 1950	Mar. 1950	Dec. 1938			
4 P_C/H	5·1	14·7	21·4	18·8	27·1	22·4	31·2	31·0	28·2			22·4	27·1	21·4	24·7	30·9	14·7	16·1	28·2			10^3
5 P_B/H	7·2	13·5	16·7	16·9	18·1	18·0	23·4	20·8	18·9			18·0	18·1	16·7	13·5	22·9	9·9	15·7	18·9			10^3
6 $L(K_1)$	5·5	9·7	19·0	12·2	40·7	21·6	19·9	27·1	31·4			47·1	93·1	48·4	21·7	31·8	25·4	41·1	88·1			10^{16}
7 $L(K_2)$	1·9	2·3	7·4	3·5	30·0	11·4	6·4	15·1	20·1			44·1	120·0	34·4	12·5	17·5	12·9	31·9	94·0			10^{22}
8 $L(K_1g)$	11·7	21·0	27·8	22·1	46·5	35·4	23·7	29·0	33·8			58·9	80·8	39·7	24·5	36·4	37·0	58·5	49·7			10^{16}
9 $L(K_2g)$	4·3	4·8	8·7	6·3	26·8	15·7	8·2	11·0	18·0			45·6	77·6	25·1	13·8	18·1	26·9	44·8	36·7			10^{22}
**10* HL/P (CK_1)	10·8	6·6	8·9	6·5	15·0	9·6	6·4	8·7	11·1	9·3	0·28	21·0	34·3	22·6	11·8	10·3	54·0	25·5	31·2	26·7	0·47	10^{12}
11 HL/P (CK_2)	3·7	1·6	3·4	1·9	11·1	5·1	2·0	4·9	7·1	4·5	0·63	19·1	4·4	16·1	8·5	5·7	27·4	19·8	33·3	16·9	0·59	10^{18}
12 HL/P (BK_1g)	16·2	15·6	16·6	13·1	25·7	19·6	10·1	13·9	17·9	16·5	0·25	32·6	44·6	23·8	18·1	15·9	37·4	37·3	26·3	29·5	0·32	10^{12}
13 HL/P (BK_2g)	6·0	3·6	5·2	3·7	1·5	8·7	3·5	5·3	9·5	5·2	0·47	25·3	42·9	15·0	10·2	7·9	27·2	28·5	19·4	22·0	0·57	10^{18}

All quantities are expressed in terms of cm and seconds as units.

Numbers in this table should be multiplied by the numbers shown in column 22.

S = Slough; W = Watheroo; H = Huancayo.

The table shows the computed magnitudes of the quantities listed in the first column. These quantities are defined in the text, and in particular in equations (6) (for L) and (10 and 13) (for P); the nomenclature is explained in §§5 to 7.

The means of the numbers in columns 1 to 9 and columns 12 to 19 are respectively listed in columns 10 and 20.

The coefficients of variation of the corresponding sets of numbers $\{\overline{(x_i-\bar{x})^2}\}^{\frac{1}{2}}/\bar{x}$ are listed in columns 11 and 21.

* Line 10 is line 6 ÷ line 4.
Line 11 is line 7 ÷ line 4.
Line 12 is line 8 ÷ line 5.
Line 13 is line 9 ÷ line 5.

Note added in proof (*7 December* 1955). Results from Port Stanley and Maui have been analyzed in the way described here. If they are included with the other results the revised figures for columns 10, 11, 20, 21 of the table are as shown below. These lend even stronger support to the conclusions reached in this paper.

	10	11	20	21
10	8·9	0·25	26·5	0·44
11	3·5	0·75	14·8	0·60
12	15·1	0·25	28·9	0·31
13	4·3	0·54	18·3	0·60

7. The best hypothesis for the rate of electron loss

The best hypothesis is now selected by the process explained in §4. The ratio HL/P for the different hypotheses is obtained from the computed values of L and P/H. The results are listed in lines 10 to 13 of table 1 and the hypotheses involved are indicated by C for Chapman and B for Bradbury, K_1 and K_2 for loss rates proportional to N and N^2 respectively, and g to indicate that the loss rate varied with height. The occasions examined are grouped into two sets, for which the sunspot numbers $\bar{R}$* were near 10 and near 80. The most satisfactory hypothesis is taken to be that for which the spread of the values, in any one group, is the least, and to help in searching for this the magnitude of the coefficient of variation† is evaluated for each group and each hypothesis and is listed in columns 11 and 21 of table 1.

It is at once evident that hypotheses involving a rate of loss proportional to N^2, i.e. those labelled K_2 or K_2g in lines 11 and 13, show a greater spread of values than those involving a rate proportional to N, and labelled K_1 or K_1g in lines 10 and 12. The difference is more noticeable in column 11, which refers to $\bar{R} \doteqdot 10$ than in column 21, which refers to $\bar{R} \doteqdot 80$. We suggest that the results indicate, at least for the smaller sunspot numbers, that the rate of electron loss is proportional to N. This result is applicable to the range of heights between 240 and about 350 km, with which we have been concerned in our calculations.

Part II. Bradbury's theory of the F_2 layer

8. Bradbury's hypothesis

In this part we propose to present a critical examination of Bradbury's hypothesis, illustrated schematically in figure 1*b*. Up to now it has not been possible to examine this hypothesis in detail, partly because the movements of the electrons in the layer were unknown, and partly because there was no estimate of the rate of electron loss. We have now obtained, in part I, an estimate of the rate of loss at different heights. We shall use it, together with a knowledge of the electron distributions in the 'mean quiet F layer' of Schmerling & Thomas (1956), and with some new observations of the F layer near sunrise, to examine how far Bradbury's hypothesis can be made self-consistent.

We shall show that it seems to be self-consistent when examined in this way, and that it leads to an estimate of the average scale height of the ionizable constituent of the atmosphere between the levels of 200 and 350 km. Finally, we shall use the results of our calculations to discuss the part played by diffusion under gravity in the F region. We shall discuss our results in part III.

9. Electron production in the F_1 layer

Let us first consider the rate of production of electrons at the peak of the production curve, supposed to be near the level of the F_1 peak of electrons. There is considerable evidence to show that the F_1 layer approximates closely to a classical Chapman layer formed at a level where the rate of electron loss is αN^2 (recombination-like) (Appleton

* Throughout this paper we have used the monthly average relative Zürich sunspot numbers.

† The coefficient of variation of a series of numbers x_i is defined as $\{\overline{(x_i-\bar{x})^2}\}^{\frac{1}{2}}/\bar{x}$. It is the ratio of the standard deviation to the mean value.

& Naismith 1935; Tremellen & Cox 1947; Allen 1948) with $\alpha \doteqdot 5 \times 10^{-9}\,\mathrm{cm^3\,s^{-1}}$ (Bates & Massey (1946), Rydbeck (1946)). If the evidence is accepted it is possible to deduce the peak rate (q_0) of production of electrons when the sun's rays are incident vertically from the standard expression

$$1{\cdot}24 \times 10^4 \{f_0 F1\,(\mathrm{Mc/s})\}^2 = N_m = \surd\{(q_0 \cos\chi)/\alpha\}. \tag{17}$$

The magnitudes of q_0, deduced in this way from the published data for Slough, together with averages of wide-world data, are shown in figure 4 plotted against mean sunspot numbers $\bar{R}$. It appears that for values of $\bar{R}$ up to 150

$$q_0 \doteqdot 280(1 + 1{\cdot}4 \times 10^{-2}\bar{R})\ \mathrm{cm^{-3}\,s^{-1}}. \tag{18}$$

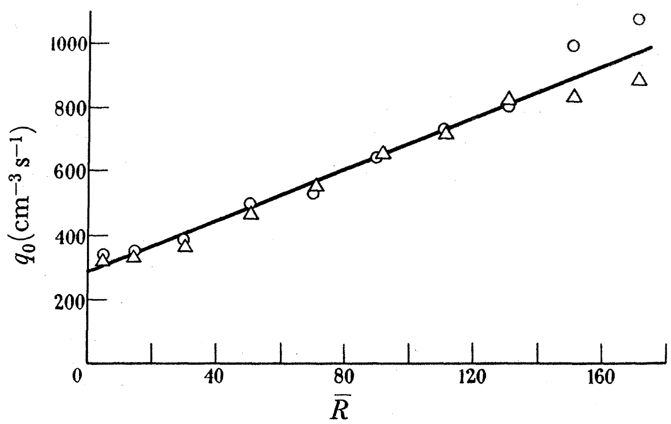


FIGURE 4. The relation between q_0 and $\bar{R}$, where q_0 is the peak rate of electron production in F_1 for $\chi = 0$, assuming $\alpha = 5 \times 10^{-9}\,\mathrm{cm^3\,s^{-1}}$ and $\bar{R}$ is the monthly average relative Zürich sunspot number. ○ Slough 1932–53; △ world averages.

In what follows we shall assume that q_0 has the value shown in figure 4. This value is proportional to the assumed magnitude of α which depends essentially on the results of somewhat unsatisfactory experiments made during eclipses.

The height of the peak of electron production in the F_1 layer is notoriously difficult to estimate. We shall here take it to be 180 km, but this is not much more than a guess. We shall return to a discussion of this height in §15 (*b*).

10. THE SOLAR CYCLE AND THE F_2 LAYER

If Bradbury's hypothesis is correct it should be possible to show that the intensity of the radiation responsible for ionizing the F_2 layer varies with the solar cycle in the same way as that responsible for the F_1 layer. Now by the argument of §4, in which the total number of electrons produced and lost in 24 h were equated to each other, it was shown that the quantity L, calculable from experiment, could be related to q_0 (the rate of production at the production peak for $\chi = 0$) and K_1 (the loss coefficient at 300 km) by equation (7), i.e. $L/P = q_0/K_1$, where P is calculable. If then K_1 is assumed to be constant throughout the solar cycle L/P should be proportional to q_0. Quantities proportional to L/P are given in table 1 for a series of places at solar epochs for which $\bar{R}$ is near 10 and near 80. The averages, and their coefficients of variation, taken from lines 12 and 13 and columns 10, 11, 21 and 22 of table 1 are listed in table 2. On the supposition that these quantities vary with $\bar{R}$ like $(1 + a\bar{R})$ the values deduced for a are shown in the last column.

The first line in the table corresponds to our previous suggestion, that the rate of loss is given by $K_1 N$. It is seen that it leads to a value of a which is consistent with the value ($1{\cdot}4 \times 10^{-2}$) deduced from observations on the F_1 layer. To this extent, therefore, Bradbury's hypothesis is satisfactory. The second line refers to a case in which Bradbury's hypothesis is used in calculating the rate of production of electrons, but their rate of loss is taken to be $K_2 N^2$. The fact that the value deduced for a is then incompatible with that deduced for the F_1 layer gives further reason for preferring the suggestion, already made in §7, that the rate of loss of electrons between the heights of 240 and 350 km is proportional to N and not to N^2.

Table 2

(The numbers in brackets represent the coefficients of variation of the quantities concerned)

hypothesis	proportional to L/P: $\bar{R} = 10$	proportional to L/P: $\bar{R} = 80$	$10^2 a$
K_1 (line 12)	17 (0·3)	30 (0·3)	1·3 (0·4)
K_2 (line 13)	5·2 (0·5)	22 (0·6)	8·7 (0·7)

11. The rate of production in the F_2 layer

If electrons are produced in the F_2 layer by the ionizing radiation which has its peak in the F_1 layer, then the rate (q) of production in the F_2 layer will depend on the average scale height (H) of the ionizable constituent. By comparing this rate with that (q_0) at the level of the peak in the F_1 layer it is possible to deduce a value for H as follows.

Equation (7) shows that

$$q_0/K_1 = L/P.$$

Now q_0 has been estimated from our knowledge of the F_1 layer and K_1 has been measured (see figure 2) to be $10^{-4}\,\text{s}^{-1}$, so that L/P can be deduced. But the quantity HL/P, computed from experimental results, is listed in line 13 of table 1, and so the magnitude of H can be deduced. The results are as follows, where the numbers in brackets represent the coefficients of variation:

For $\bar{R}$ near 10:

$$q_0 = 320 \quad HL/P = 16 \times 10^{12}\,(0{\cdot}25),$$

giving

$$H = 46\,(0{\cdot}25)\ \text{km}.$$

For $\bar{R}$ near 80:

$$q_0 = 600 \quad HL/P = 29 \times 10^{12}\,(0{\cdot}32),$$

giving

$$H = 44\,(0{\cdot}32)\ \text{km}.$$

In considering the significance of these figures the following points should be noted:

(*a*) The magnitude deduced for H is inversely proportional to q_0, which is itself proportional to α in the F_1 layer. It has already been mentioned that the magnitude of this quantity is somewhat uncertain. If the value 8×10^{-9} recently suggested by Minnis (1955) were used we should deduce $H = 28$ km.

(*b*) In the computations leading to the figures of table 1 the lower boundary of the unit column under consideration was taken to be at 240 km. In the corresponding theory of §5 (*b*) it was taken to be a distance $1{\cdot}5H$ above the level of the production peak for vertically incident radiation. Now that we have deduced that $H \doteqdot 45$ km these two values are seen to be consistent if we assume that the production peak is near 180 km.

(*c*) The magnitude of K_1 was determined from observations made at night, whereas in estimating the total loss it is the day-time contribution which is of major importance, since the rate of loss is proportional to the electron density (N).

12. Observations near sunrise

One of us (C. S. G. K. S.) has recorded $h'(f)$ curves at frequent intervals through the period of sunrise at Cambridge, for a series of days between April 1954 and April 1955. $N(t)$ curves were plotted from the records to show how the electron density (N) depended on the time (t) at a series of different heights. The details of this work will be presented in a separate paper, but we shall here make use of the preliminary results to deduce the rate (q) of production of electrons near sunrise at a height of 320 km. By comparing this rate with q_0 we shall make another, and independent, estimate of H.

The rate of increase of electron density at a height (h) is given by the equation

$$\mathrm{d}N/\mathrm{d}t = q - KN - M, \tag{19}$$

where K represents the loss coefficient and M represents the effect of movements. If the only movements are in the vertical direction, with an upward velocity w, then $M = \mathrm{d}(Nw)/\mathrm{d}h$. Suppose now observations are made just before layer sunrise at the height considered and at a certain time soon after layer sunrise. Then, by subtracting two equations like (19), one for each time (subscripts 1 and 2), we find

$$q_2 = (\mathrm{d}N/\mathrm{d}t)_2 - (\mathrm{d}N/\mathrm{d}t)_1 + [K(N_2 - N_1) + M_2 - M_1 + q_1]. \tag{20}$$

If the terms in the square brackets could be neglected it would be possible to determine q_2 from observations of $\mathrm{d}N/\mathrm{d}t$. Reasons will now be given which suggest that the terms mentioned are probably negligible, under certain conditions.

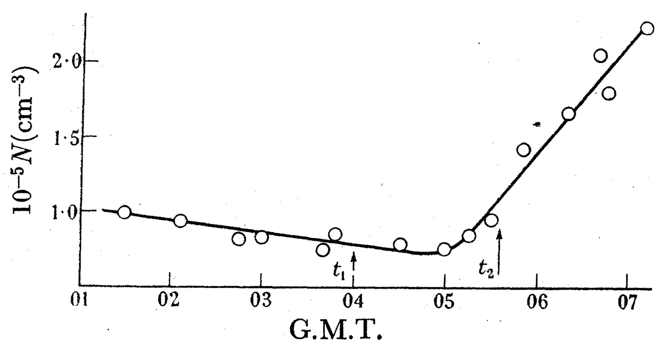


Figure 5. Electron density N as a function of time at a height of 320 km on 2 April 1954.

We shall consider $N(t)$ curves, of the type shown in figure 5, obtained in summer for a height of 320 km. It is noticeable that there is a marked change of the order of 20 cm^{-3} s^{-1} in the slope of the curve shortly before ground sunrise. The average magnitude

$$[(\mathrm{d}N/\mathrm{d}t)_2 - (\mathrm{d}N/\mathrm{d}t)_1]$$

of this change, for 10 days between March and September 1954, was 19 ± 1 cm^{-3} s^{-1}. On the same days the average magnitude of $K(N_2 - N_1)$ calculated by putting $K = 7 \times 10^{-5}$ s^{-1}, appropriate to 320 km, was 3·5. The term $K(N_2 - N_1)$ can therefore be neglected in the equation.

It is more difficult to justify the neglect of the movement term (M_2-M_1). It is, however, noticed that the rapid change in the slope of the $N(t)$ curve occurs, at all seasons, during the hour before ground sunrise, and it appears probable that it is caused by a sudden increase of q, rather than by a sudden change of M occurring just at this time.

We now select the first time (t_1) just before sunrise on the layer, and write $q_1 = 0$, and also assume that the other terms in the square brackets are negligible. The time (t_2) is taken at ground sunrise, when the solar zenith angle χ is 90° and q_2 is calculated from the measured magnitudes of $(\mathrm{d}N/\mathrm{d}t)_1$ and $(\mathrm{d}N/\mathrm{d}t)_2$ by the use of equation (20). The mean value found for $q(\chi = 90)$ at a height of 320 km on days in the period March to September 1954, was $19\pm1\,\mathrm{cm^{-3}\,s^{-1}}$.

We now relate this magnitude of q to the magnitude (q_0) at the peak of the production curve when $\chi = 0$. The relation is given by the expression, due to Chapman,

$$q = q_0 \exp\left\{1+\frac{h_0-h_1}{H} - Ch(\chi, H)\exp\left(\frac{h_0-h}{H}\right)\right\}. \tag{21}$$

Because we are concerned with times near sunrise it is necessary here to use the function $Ch(\chi, H)$, introduced by Chapman (1931 *b*) to allow for the curvature of the earth. Its magnitude depends on the scale height assumed for the gas which absorbs the ionizing radiation. If we insert the values $q = 19\pm1$, $q_0 = 340$ (appropriate to the epoch 1954) $h = 320$, $h_0 = 180$, into equation (21) we find, by trial, that it is satisfied for values of H between 40 and 45 km. This range of values is not inconsistent with that obtained in §11 by considering the total production and loss of electrons throughout a day.

13. Diffusion under gravity in the F_2 layer

If the Bradbury hypothesis is accepted it is now possible to use the equation

$$\mathrm{d}N/\mathrm{d}t = q-KN-M \tag{19}$$

to determine the magnitude of M at any height and time. For this purpose the magnitudes of N and $\mathrm{d}N/\mathrm{d}t$ are taken from the experimental results, q is calculated from equation (21) with H put equal to 45 km and K is obtained from figure 2. The detailed results are not yet available, but their order of magnitude is already clear. The maximum magnitude deduced for M on any one day at a distance of about 50 km below the electron peak was of the order $100\,\mathrm{cm^{-3}\,s^{-1}}$ near midday and $40\,\mathrm{cm^{-3}\,s^{-1}}$ near midnight*.

The movements whose effects are represented by M are presumably caused jointly by the bodily movements of the atmosphere (if any), by electromagnetic forces, and by diffusion under gravity. We shall write

$$M = M_1 + M_D, \tag{22}$$

where M_D represents the result of diffusion under gravity and M_1 the result of other movements. The magnitude of M_1 is difficult to estimate without considerable speculation, but the magnitude of M_D seems to be well established from standard diffusion theory. Hulburt (1928) and Ferraro (1945) have shown how it depends on the shape of the electron distribution, the number density of neutral particles and the scale height of the composite

* The mean magnitude of M near midnight must, on our argument, be zero, since it was assumed to be zero when the values of K were deduced in § 3.

atmosphere at the level considered. The smaller the number density of neutral particles the greater the rate of diffusion. If therefore we could set an upper limit to the magnitude of M_D we could deduce a lower limit for this number density in the F_2 layer. This is what we now attempt to do.

We have already stated that maximum values of M have been found approximately. Now it might be that the movements produced by diffusion under gravity (M_D) and by other forces (M_1) were both larger than M but of opposite signs. All theories agree, however, in suggesting that M_1 is oscillatory and that it has different signs at different times of day and night, whereas M_D has the same sign at all times for the cases we shall consider. Under these circumstances it seems safe to say that, however large the oscillatory term M_1 may be, M_D cannot be greater than the maximum value of M. We shall therefore take the computed magnitude of M to be an upper limit to M_D.

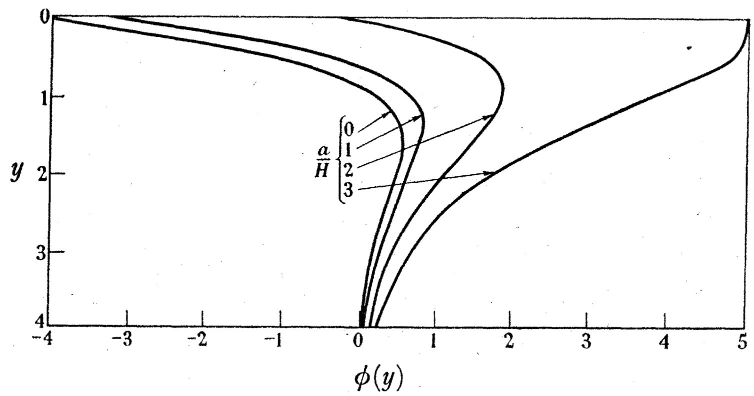


FIGURE 6. The function $\phi(y) = \exp(-y)\{a^2/H^2 - 4 + 6y - y^2\}$ of equation (26) for different values of a/H.

We now turn to consider the expected magnitude of M_D. Ferraro (1945) has shown that it is given by an expression which may be written in the form

$$M_D(h) = -\frac{b}{n_1}\sin^2 I \exp\left\{\frac{h-h_1}{H}\right\}\left\{\frac{d^2N}{dh^2} + \frac{3}{2H}\frac{dN}{dh} + \frac{N}{2H^2}\right\}, \tag{23}$$

where $b \doteqdot 10^{19}\,\text{cm}^{-1}\,\text{s}^{-1}$ is a constant derived from the theory of diffusion,

n_1 = the number density of the composite atmosphere at a reference level h_1,

I = the angle between the horizontal and the earth's magnetic field,

H = the scale height,

N = the number density of electrons,

h = the height.

Except at Huancayo the electron distribution profiles with which we shall have to deal are approximately parabolic, as given by

$$N = N_m\{1 - (h_m - h)^2/a^2\}. \tag{24}$$

Under these circumstances the following expression may be derived from equation (23):

$$M_D(y) = -(bN_m/2n_m a^2)\,\phi(y)\sin^2 I, \tag{25}$$

where N_m, n_m and h_m represent values at the peak of the electron layer

$$y = (h_m - h)/H$$

and

$$\phi(y) = \exp(-y)\{(a^2/H^2) - 4 + 6y - y^2\} \tag{26}$$

The function $\phi(y)$ is shown, for different values of the ratio a/H, in figure 6.

Equation (25) was then used in the following way. $M_D(y)$ was given the value 100 at midday and 40 at night appropriate to $y = 1$, i.e. to a distance of about 45 km below the height (h_m) of the electron peak, and the parameters (a, h_m and N_m), which specify the shape of the layer, were determined from the 'mean quiet F layers'. The equation then provided a value for n_m, the number density of neutral particles at the height h_m. The data for different occasions at Slough and Watheroo are shown in table 3. According to our calculation the values of n_m are to be considered as minimum values. They will be discussed in § 15 (c).

Table 3. F_2 layers observed at Slough and Watheroo

	midday				midnight			
date	a (km)	h_m (km)	$10^{-5} N_m$ (cm^{-3})	$10^{-8} n_m$ (cm^{-3})	a (km)	h_m (km)	$10^{-5} N_m$ (cm^{-3})	$10^{-8} n_m$ (cm^{-3})
				Slough				
Sept. 1953	40	270	4	8	75	360	1·2	3·7
Mar. 1950	70	320	13	17	115	400	3·5	8·7
Dec. 1953	35	250	4	9	60	320	1·3	4·5
July 1950	70	330	6	8	105	380	4·8	1·3
				Watheroo				
Mar. 1944	70	320	7·7	9	60	330	1·7	5·7
Mar. 1939	150	360	13·6	14	100	380	3·2	8·5
Dec. 1944	75	365	7·2	9	65	360	3·0	9·5
June 1940	50	320	8·4	16	80	365	1·3	3·7

The parameters (a, h_m, N_m) of equation (24) which describe the electron distribution at the times shown, and the magnitude of n_m, the minimum number density of neutral particles at the height h_m, deduced from them.

PART III. GENERAL DISCUSSION

14. The rate of electron loss

In parts I and II we have outlined mechanisms of electron production and loss in the F region which seem to be fairly self-consistent when they are tested in various ways which do not call for detailed assumptions about the movements in that region. In this part we summarize and discuss our conclusions.

First it was suggested, in § 7, that the rate of electron loss between heights of 240 and about 350 km is proportional to N and not to N^2. This conclusion rests mainly on the arguments of § 7 concerning the total number of electrons lost and gained in 24 h in years of small sunspot number. It involves several assumptions, for example, that the layer, whether Chapman or Bradbury, is produced by a monochromatic ionizing radiation; that the nature of the atmosphere, including its average scale height, is the same at different places, seasons and solar epochs; that the loss coefficient is likewise constant; and that the total transport of electrons back and forth over the top and bottom of a column is on the whole small when integrated over 24 h (see § 4). The fact that some, or all, of these assumptions are invalid may account for the spread of the figures in table 1 and particularly those which refer to large sunspot numbers. It seems reasonable, however, to make the assumptions mentioned in order to arrive at a description which might be even approximately correct.

Next it was suggested, in § 3, that the electron-loss coefficient (K) decreases upwards as shown in figure 2. This conclusion rests on night-time measurements which give consistent results at two places and all seasons near sunspot maximum. The assumption is made that, on these occasions, movements were unimportant. There might, of course, have been important movements together with a quite different distribution of the loss coefficient.

When we consider the loss rate outside the range of heights 240 to 350 km there is reason to think that the circumstances are different. Thus there is evidence that, at the peak of the F_1 layer, at a height near 180 km, the rate of electron loss is about $5 \times 10^{-9} N^2$, and that it does not change much with height.

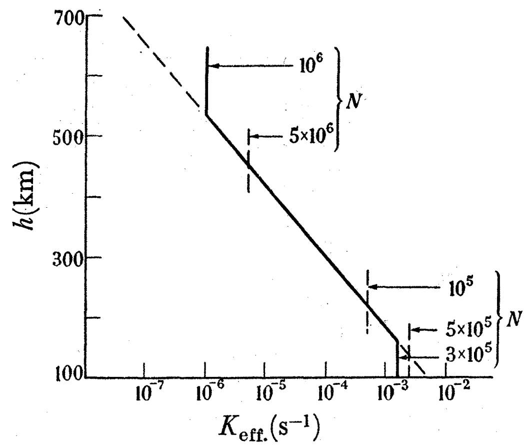


FIGURE 7. Suggested variation of loss coefficient with height. The possible forms of the variation at the two ends of the curve depend on the electron density (N) as explained in the text. The solid line might represent the variation for an actual layer.

These indications from experiment are closely parallel to the suggestions made by Bates & Massey (1948) on theoretical grounds. They concluded that loss coefficients could only be as great as those measured if dissociative recombination was important. In the higher parts of the atmosphere they supposed atomic oxygen was the ionizable constituent and that there were molecules XY (which might possibly be molecular oxygen O_2) to which positive charges could be transferred from the atomic oxygen ions. The processes envisaged were then

$$\text{production} \quad O + h\nu \rightarrow O^+ + e, \tag{a}$$

$$\text{loss} \quad \begin{cases} O^+ + XY \rightarrow O + XY^+, & (b) \\ XY^+ + e \rightarrow X + Y. & (c) \end{cases}$$

Where the molecules XY are scarce the process (b) would be rate-determining, and the rate of loss of electrons would be proportional to $n(O^+)\, n(XY)$, i.e. to $n(e)\, n(XY)$. This would correspond to our suggestion that, above 240 km, the rate of loss is proportional to N (i.e. $n(e)$) and its decrease upwards would correspond to the upwards decrease of $n(XY)$. Lower down it was suggested that there would be enough molecules XY to make $n(XY^+) \doteqdot n(O^+)$ so that the rate of loss, determined now by process (c), would be proportional to $[n(e)]^2$ and only slightly dependent on height. This would correspond to the usual suggestion for the F_1 layer near 200 km.

Other theoretical reasons suggest that the electron-loss coefficient does not continue to decrease with height at the greatest heights. It has, for example, been pointed out that, at the greatest heights, radiative recombination must ultimately be the predominant process, with a rate of loss given approximately by $10^{-12} N^2$ (Bates, Buckingham, Massey & Unwin 1939). If we write $N \doteqdot 10^6$ then the loss process described by equation (1) will be as rapid as that caused by radiative recombination at a height of about 550 km.

If the rate of loss is represented, at all levels, by $K_{\text{eff.}} N$, then, on these ideas, $K_{\text{eff.}}$ is given by equation (1) at intermediate levels and by $5 \times 10^{-9} N$ low down and $10^{-12} N$ high up. It is shown in figure 7 for a series of different values of N. Thus if near 200 km the electron density were 3×10^5 (a reasonable day-time value for the F_1 electron peak) and near 500 km it were 10^6 (a reasonable daytime value for the F_2 peak) the curve drawn in a continuous line would approximately represent the loss rate. The implications of this figure are of interest in considering the possibilities for the bifurcation of the F layer.

15. Molecular densities in the upper atmosphere

(a) *The average scale height of the ionizable gas*

In part II a critical examination was made of Bradbury's hypothesis and in § 11 it was shown that the average scale height of the ionizable gas, between heights of 200 and 350 km would have to be of the order of 45 km if the electron production and loss were to balance throughout 24 h. In § 12 some completely different observations, made near sunrise, led to the conclusion that this average scale height was between 40 and 45 km. In what follows we shall suppose that the most acceptable value is about 45 km.

It is interesting to compare this scale height with recent suggestions made by Bates (1954), who deduced the number densities of O and N_2, at different heights, by extrapolating the results of experiments with rockets. He called the resulting model of the atmosphere the R model to distinguish it from the previously used G model which was based on the results of experiments conducted on the ground. In the R model the temperature of the F region is less than in the G model, and the scale height is correspondingly smaller. The R model is illustrated in figure 8, from which it can be seen that between heights of 200 and 400 km the average scale height of O is about 50 km and that of N_2 is about 30 km.

It has often been suggested that the ionizable constituent in the F region is O, and it is interesting to note that over this same range of heights our value of 45 km for the scale height is not very different from that of the suggested R model. If we had taken a larger value for α in the F_1 layer, as suggested by Minnis (1955), we should have found even smaller values for the scale height.

(b) *The height of the peak of production*

In § 9 it was assumed arbitrarily that the peak of electron production when $\chi = 0$ was at a height $h_0 = 180$ km. This seems to be a reasonable assumption but, since our knowledge is so uncertain, it is well to ask how our calculations would be altered if another assumption were made. It is found that, if $h_0 = 200$ km, then H, the scale height of the ionizable gas, is 35 km, and if $h_0 = 160$ km, then $H = 65$ km. These values of H are deduced both by the method of integration over the day (§ 11) and from the observations near sunrise (§ 12).

With our assumption that $h_0 = 180$ km, and the corresponding deduction, that $H = 45$ km, the lower level 240 km, at which the numerical integration of § 6 was terminated, is approximately 1·5 scale heights above h_0. The value $z_1 = 1{\cdot}5$, used in § 5 (*b*), is thus sufficiently nearly correct.

(c) The upwards decrease of the loss coefficient

In § 14 it was explained that, on the theory of Bates & Massey (1948), the loss coefficient K would, under certain circumstances, decrease with height in the same way as the number-density of oxygen molecules O_2. Nicolet & Mange (1954) have suggested that, under the joint effects of photo-dissociation and diffusion, O_2 would be distributed in the F region

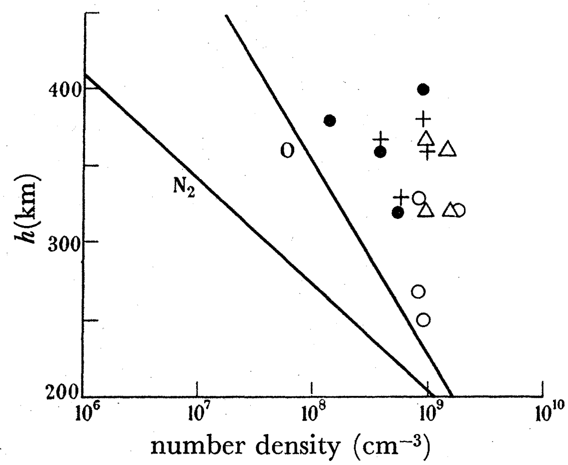


FIGURE 8. The lines represent the number densities of N_2 and O according to the R model of Bates (1954). The points represent minimum values of the number density deduced from the observations and the theory of diffusion. ○ Slough (day); ● Slough (night); + Watheroo (night); △ Watheroo (day).

with its normal scale height. The appropriate scale height would not be much different from that for N_2 which, on the R model of Bates, shown in figure 8, is about 30 km. The experimental values of the loss coefficient, shown in figure 2, appear, however, to decrease upwards like $\exp(-h/50)$ (see equation (1)), i.e. with a 'scale height' of 50 km. This is more nearly that of O in the R model, and is, within the limits of experimental accuracy, the same as the 'scale height' of the ionizable gas.

(d) Diffusion under gravity

In § 13 we used considerations of diffusion under gravity to derive minimum values for the number density of neutral particles at a series of given heights. These were listed as n_m in table 3 and are plotted in figure 8. The theory of Ferraro (1945), on which our calculations were based, assumed that the ionizable constituent was the only one present, so that it will be most nearly correct to compare our values with Bates's curve for O. There is a significant difference between the two, and it appears that our values, which are minimum values, are about five times those of Bates. We can put this point another way, and say that, if the R model were correct we should expect the movement term to be about five times greater than the one we have calculated from the experimental results. We thus agree with the remarks that Bates (1954) and Martyn (1955 *a*, *b*) have previously made, that it seems as though diffusion under gravity would be too large on the R model to escape observation.

Although there is this significant discrepancy between our results and the deductions of Bates it does, nevertheless, appear remarkable that, to a first order, they are roughly the same. Indeed, it almost seems that the level of the layer had adjusted itself so that the diffusion term would not be evident from the calculation. If, for example, the layers listed for Slough in table 3 had been formed by day at the levels where they are found at night (i.e. about 2 scale heights higher) the diffusion terms would have been about 10 times greater and they would certainly have been noticeable in our results. It is a little remarkable that the layer seems to be just low enough for the effects of diffusion not to be measurable.

These results lead us to suggest that possibly when the F_2 layer is high the level of its peak is determined by the diffusion process itself. Suppose, for example, that other processes tended to form a layer with its peak one or two scale heights above the observed peak. Then at these greater levels diffusion would be rapid, and since diffusion under gravity always tends ultimately to lower the peak of a layer, that peak would fall to a level where diffusion was no longer the dominant effect. But that is just where, in fact, it is found to be.

It is interesting, in connexion with these ideas, to notice that those places where the peak of electron density is high are near the geomagnetic equator, where the earth's magnetic field suppresses vertical diffusion, as indicated by the term $\sin^2 I$ in equation (23). At Huancayo, for example, in December 1938, the electron peak was at a height of about 450 km, and the magnitude of M calculated from the experimental results by using equation (19) was about 100. If the same distribution of electrons had been produced at moderate latitudes, where $\sin^2 I \doteqdot 1$, equation 25* shows that the magnitude of M_D one scale height below the maximum would have been about 10^4, if the atmosphere is as suggested by Bates. The diffusion processes would clearly prevent the distribution frequently produced at Huancayo ever being produced, for example, at Slough.

Since the effect of the earth's magnetic field in suppressing diffusion is proportional to $\sin^2 I$ it seems reasonable to suggest that it is this suppression which is primarily responsible for the difference in the behaviour of the ionosphere at low and at medium latitudes, and for the occurrence of the 'geomagnetic anomaly'. In this connexion it is interesting to note that diffusion would produce important changes in about 1 h at a level of 300 km and in a considerably shorter time higher up.

16. General conclusion

Athough it is realized that movements play a predominant part in determining the shape of the F_2 layer, the purpose of this paper has been to devise methods of calculation in which their effects are small. In the investigation of the rates of production and loss the effects have then been neglected. In each case some reason has been given for thinking that the neglect might be less serious for our method of calculation than for methods used previously and the fact that our calculations lead to fairly consistent results is encouraging. Although it is impossible to claim that our suggestions about the rates of production and loss of electrons are the only ones which would be consistent with the facts, it does seem that it might be worth while to investigate their implications more fully. This we hope to do.

* The electron distribution observed at Huancayo was more nearly linear than parabolic, and an expression appropriate to this distribution was used, instead of equation 25, in the calculation. The orders of magnitude would not have been altered if a parabola had been adjusted to fit the observations approximately and equation (25) had been used.

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